Computing Three Dimensional Fluids

Dennis Parnell Sullivan

2014 Balzan Prize for Mathematics (pure/applied)

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Period: 2016-2018

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In order to compute fluid motion, any fluid model must be discretized in terms of finitely many parameters. Discretizing space by dividing it into cells was Poincaré's starting point when he invented topology to study qualitative dynamical systems just over one hundred years ago. In the middle of the twentieth century, great advances were made in algebraic topology, which is also based on these cells. These advances were related to the algebraic products that are involved in the discretization process for the nonlinear term of the fluid models.

When discretizing, certain algebraic symmetry in the ideal models is broken. This loss of symmetry is correctable by an elegant hierarchy of corrections based on these algebraic topology advances. These corrections are similar but not exactly the same as the Feynman diagrams used in the algorithms to compute physical effects in quantum theories.

Sullivan and his student colleagues have long been engaged in understanding the corrections and building theoretical algorithms for fluid computations based on these conceptual ideas. Their work led to the revelation that different ways of writing the ideal model which are equivalent at the ideal level are inequivalent at the discrete level.

There is coherence, however, if one allows for the extended sequence of corrections alluded to above. Systematically testing the various algorithms in terms of their extended corrections would hence prove beneficial in connection with known fluid data. With the second part of his Balzan Prize, Sullivan would take up a project that he has wanted to carry out for some time, that is to say, initiating the practical part of this theoretical work, which is essentially complete.

The project will primarily be based at Stony Brook University, with parts being carried out at the Graduate Center of the City University of New York.