Dynamical Systems, Chaotic Behaviour – Uncertainty, Linear Cocyles and Lyapunov Exponents

Jacob Palis

2010 Balzan Prize for Mathematics (pure and applied)

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Period: 2011-2015
Websites: http://www.impa.br/opencms/en/eventos/store_2016/evento_1606; http://www.impa.br/opencms/pt/eventos/store_old/evento_1203?link=2; http://www.impa.br/opencms/pt/eventos/store_old/evento_1305?link=2; http://www.impa.br/opencms/pt/eventos/store_old/evento_1502

Jacob Palis is a Professor at the Istituto de Matemática Pura e Aplicada (IMPA) in Rio de Janeiro. The objective of his research project is to involve scientists from different regions of the world, in particular, talented young mathematicians. One of its main goals is to advance a Global Conjecture, stated by Palis twenty years ago, concerning the finiteness of the number of attractors for typical dynamics in closed manifolds. Other important topics were linear cocycles and Lyapunov exponents. Coordinated together with Fields Medalist Jean-Christophe Yoccoz (Collège de France) at the Instituto de Matemática Pura e Aplicada, IMPA, Rio de Janeiro, Brazil, the project has set out to study (and hopefully prove) a set of conjectures for dynamical systems that leads to a global perspective in this important branch of mathematics.

The Research Project was scheduled to take place from 2011 to 2015, with part of the funds supporting the activities of young researchers at IMPA in research on *Dynamical Systems, Chaotic Behaviour-Uncertainty*. Three Palis-Balzan Symposia on Dynamical Systems also took place during that period. The first was held at IMPA in 2012, and the following two Symposia took place at the Institut Henri Poincaré in

Paris in 2013 and 2015. These symposia were designed to review advances and to stimulate further progress along the lines of the research project.

The Third Palis-Balzan Symposium on Dynamical Systems was organized by IMPA and Collège de France, with support from CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nivel Superior), CNRS, ANR, IMPA (Associaçiao Instituto Nacional de Matemática Pura e Aplicada), Institut Henri Poincaré, Mairie de Paris, Université Paris 13 and Université Paris-Sud 11. It aimed at promoting research at the highest level in the area of dynamical systems, with the effective participation of outstanding groups of researchers at the world level. The Symposia also aimed at putting doctoral students and young researchers in touch with the best of what is produced worldwide on the above and related topics, disseminating recent results and providing high level international scientific exchange of ideas and results. In particular, it stimulated the further development of the Brazilian group in the area, which has been increasingly asserting itself internationally.

The Organizing Committee of the Third Syposium was comprised of Sylvain Crovisier (CNRS), Jacob Palis (IMPA), Carlos Matheus Santos (CNRS) and Jean-Christophe Yoccoz (Collège de France); the members of the Scientific Committee were Artur Avila (IMPA and CNRS), Sylvain Crovisier (CNRS), Michael Lyubich (SUNY at Stony Brook - USA), Welington de Melo (IMPA), Carlos Gustavo Moreira (IMPA), Jacob Palis (IMPA), Enrique Pujals (IMPA), Carlos Matheus Santos (CNRS), Marcelo Viana (IMPA) and Jean-Christophe Yoccoz† (Collège de France).

The following papers were presented at the Third Palis-Balzan Symposium on Dynamical Systems: Artur Ávila (IMPA and CNRS), Continuity of Lyapunov exponents for random matrix products; Lucas Backes, Continuity of Lyapunov Exponents for Fiber-Bunched Cocycles; Jairo Bochi (PUC-Chile), Ergodic optimization and prevalence; Ricardo Bortolotti (UFPE), Physical measures for certain partially hyperbolic attractors on 3-manifolds; Christian Bonatti (Université de Bourgogne - Dijon), Singular hyperbolicity and star flows; Gonzalo Contreras (CIMAT), The C2 Mañé's Conjecture on Surfaces; Lorenzo Diaz (PUC-RJ), The entropy spectrum of Lyapunov exponents in non-hyperbolic skew-products; Elise Goujard, Counting closed geodesics on flat surfaces; Federico Hertz (University Park), Random Dynamics and a formula for Furstenberg, Kullback-Ledrappier Entropy; Sergio Ibarra (UFRJ), On the Lagrange and Markov Dynamical spectra for flows and geodesic flows; Vadim Kaloshin (University of Maryland), Stochastic Arnold diffusion of deterministic

systems; Juan Rivera-Letelier (PUC-Chile), Sensitive dependence of geometric Gibbs measures of one-dimensional maps; Michael Lyubich (SUNY at Stony Brook), On selected problems in Holomorphic Dynamics; Marco Martens (SUNY at Stony Brook), Renormalization and Symmetry; Carlos Gustavo Moreira (IMPA), On the fractal geometry of horseshoes in arbitrary dimensions; Liviana Palmisano (IMPAN, Poland), On circle maps with a flat interval and Cherry flows; Vilton Pinheiro (UFBA), Finiteness of attractors for piecewise C2 maps of the interval; Enrique Pujals (IMPA), On a conjecture of Charles Tresser about surfaces diffeomorphisms in the boundary of chaos; Rafael Potrie (Centro de Matemática – Univ. de la República), The classification problem for partially hyperbolic diffeomorphisms; Martin Sambarino (Fac. de Ciencias del Uruguay), Attracting circloids and entropy; Weixiao Shen (National University of Singapore), Hausdorff dimension of the graphs of the classical Weierstrass functions; James Tanis (Collège de France), Effective equidistribution of horocycle maps; Damien Thomine (Université Paris-Sud), Potential kernel, hitting probabilities and limit theorems for Abelian extensions of periodic dynamical systems; Masato Tsujii (Kyushu University), On the error term of the Prime Orbit Theorem for expanding semi-flows; Marcelo Viana (IMPA), Cr-density of hyperbolicity (non-uniform) among partially hyperbolic diffeomorphisms; Amie Wilkinson (University of Chicago), Mixing properties of the Weil-Petersson geodesic flow; Lan Wen (Peking University), The problem of characterizing structural stability revisited.

Another International Conference on Dynamical Systems was held from 4 to 8 July 2016 in Rio de Janeiro. Jacob Palis (IMPA) acted as Coordinator, and the other members of the Organizing Committee were Artur Avila (IMPA and CNRS), Sylvain Crovisier (CNRS), Carlos Matheus Santos (CNRS Paris, France), Marcelo Viana (IMPA) and Jean-Christophe Yoccoz† (Collège de France).

Invited speakers included: Lucas Backes (Universidade Federal do Rio Grande do Sul), Continuity of Lyapunov Exponents is Equivalent to Continuity of Oseledets Subspaces; Christian Bonatti (Université de Bourgogne - Dijon), (Multi)singular hyperbolic structures; Milton Cobo (Universidade Federal do Espirito Santo), Wandering intervals for affine perturbations of the Arnoux-Yoccoz family; Javier Correa (UFRJ), Transitivity of covering maps from the torus without resonance; Sylvain Crovisier (CNRS-France), Finiteness of measures maximizing the entropy for surface diffeomorphisms; Romain Dujardin (Université Paris Est Marne La Vallée), Non-density of stability for holomorphic mappings on Pk; Fayad Bassam (Institut de Mathématiques Jussieu), Lebesgue spectrum for area preserving flows

on the two torus; Nicolas Gourmelon (Université de Bordeaux), Projectively Anosov diffeomorphisms of surfaces; Pierre-Antoine Guihéneuf (Universidade Federal Fluminense), Physical measures of C^1 generic diffeomorphisms: what see the discretizations; Carlos Gustavo Moreira (IMPA), On the fractal geometry of horseshoes in arbitrary dimensions; Kei Irie (Kyoto University), A C?-closing lemma for three-dimensional Reeb ows via embedded contact homology; Vadim Kaloshin (University of Maryland), On deformational spectral rigidity of convex symmetric planar domains; Alejandro Kocsard (Universidade Federal Fluminense), On the dynamics of mininal homeomorphisms of T 2; Patrice Le Calvez (Institut de Mathématiques de Jussieu), Orbit forcing theory for surface homeomorphism; Fernando Lenarduzzi (IMPA), The ergodicity of the restricted three-body problem: the Hénon-Devaney map; Mikhail Lyubich (SUNY at Stony Brook), On the dynamics of dissipative complex Henon maps; Karina Marin (IMPA), Lyapunov exponents of partially hyperbolic volume-preserving maps with 2-dimensional center bundle; Welington de Melo (IMPA), Rigidity of critical circle maps; Vilton Pinheiro (Universidade Federal da Bahia), On the flexible concept of ergodicity; Tali Pinsky (Tata Institute of Fundamental Research), A topological approach to the Lorenz equations; Mauricio Poletti (IMPA), Simple Lyapunov spectrum for certain linear cocycles over partially hyperbolic maps; Rafael Potrie (Centro de Matemática – Universidad de la República), Entropy rigidity for surface group representations; Enrique Pujals (IMPA), On the Cr-typicality of coexistence of infinitely many sinks; Sergio Romaña (Universidade Federal do Rio de Janeiro), The Hausdorff Dimension for Geometric Lorenz Attractor; Martin Sambarino (Faculdad de Ciencias del Uruguay), Rotation set and entropy for attracting anular continua; Michele Triestino (Universidade Federal Fluminense), Markov partitions for groups of circle diffeomorphisms; Ricardo Turolla Bortolotti (Universidade Federal de Pernambuco), Physical measures for certain partially hyperbolic attractors on 3-manifolds; Jiangong You (Nanjing University), On Lyapunov exponent and Avila´s acceleration of quasi-periodic Schrödinger cocycles.

For the programmes of the first two Palis-Balzan Symposia, the reader is referred to previous editions of the *Overview* of the Balzan research projects and the Balzan website: http://www.balzan.org/en/prizewinners/jacob-palis/research-project-palis.